# AN AXISYMMETRIC TEMPERATURE FIELD IN A SOLID CIRCULAR CYLINDER

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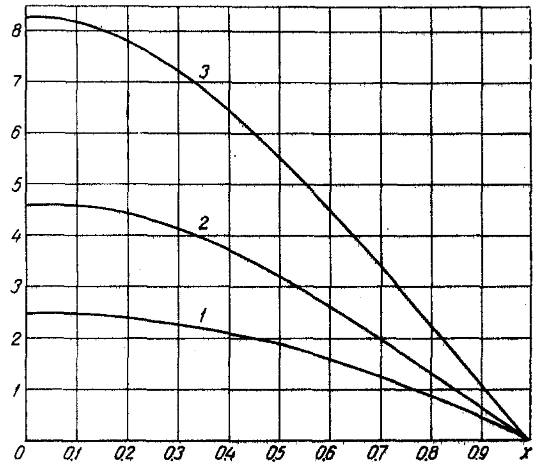
The complete theory of heat conduction and all the basic problems associated with the cylindrical shape have been set out in [1].

The present article represents a certain departure from the traditional methods of determining temperature fields.




Variation of the coefficients of the series (8): 1) $\left[\alpha_1 - \alpha_0 \left(\frac{x}{2}\right)^2\right] \cdot 10$; 2) $\left[\alpha_2 - \alpha_1 \left(\frac{x}{2}\right)^2 + \frac{\alpha_0}{(2!)^2} \times \left(\frac{x}{2}\right)^4\right] \cdot 10^2$; 3) $\left[\alpha_3 - \alpha_2 \left(\frac{x}{2}\right)^2 + \frac{\alpha_1}{(2!)^2} \left(\frac{x}{2}\right)^4 - \frac{\alpha_0}{(3!)^2} \left(\frac{x}{2}\right)^6\right] \cdot 10^3$.

Let u be the temperature satisfying the unsteady heat conduction equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} - \frac{\partial u}{\partial \tau} = 0, \tag{1}$$

where $\tau = at$ is the reduced time.

The cylinder radius r varies from 0 to b. We introduce the dimensionless variable $x = r/b$; then (1) takes the form

$$\frac{1}{x} \frac{\partial}{\partial x} \left(x \frac{\partial u}{\partial x}\right) + \Delta u = 0, \tag{2}$$

where

$$\Delta = b^2 \left( \frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial \tau} \right). \tag{3}$$

As may be verified by direct substitution, a solution of (2) will be

$$u = \sum_{k=0}^{n} \frac{(-1)^k}{(k!)^2} \left(\frac{x}{2}\right)^{2k} \Delta^k S. \tag{4}$$

In (4) n is a nonnegative integer, and S is an arbitrary analytic function of two variables (z and $\tau$). If we require that S satisfy the equation

$$\Delta^n S = 0, \tag{5}$$

then the series (4) will be finite.

We will prove the convergence of (4) as $n \to \infty$. Since S is analytic, in its region of definition there is a number $l$ such that $|\Delta^l S|$ is greatest.

We construct the series

$$|\Delta^l S| \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left(\frac{x}{2}\right)^{2k}, \tag{6}$$

of which each term is larger than each term of series (4). The latter, however, converges absolutely, since under the summation sign we have a Bessel function of imaginary argument ($I_0(x)$). Therefore, series (4) converges even more rapidly, which was to be proved.

Let $q(z, \tau)$, an arbitrary analytic function of its own coordinates, be the assigned value of temperature on the lateral surface of the cylinder $r = b$ ($x = 1$).

We define the function S from (4) in the form of a series,

$$S = \sum_{k=0}^{\infty} \alpha_k \Delta^k q, \tag{7}$$

where $\alpha_k$ are constants as yet unknown.

Substituting (7) into (4), we obtain

$$\begin{aligned} u = {} & \alpha_0 q + \left[\alpha_1 - \alpha_0 \left(\frac{x}{2}\right)^2\right] \Delta q + \\ & + \left[\alpha_2 - \alpha_1 \left(\frac{x}{2}\right)^2 + \frac{\alpha_0}{(2!)^2} \left(\frac{x}{2}\right)^4\right] \Delta^2 q + \\ & + \left[\alpha_3 - \alpha_2 \left(\frac{x}{2}\right)^2 + \frac{\alpha_1}{(2!)^2} \left(\frac{x}{2}\right)^4 - \frac{\alpha_0}{(3!)^2} \left(\frac{x}{2}\right)^6\right] \Delta^3 q + \ldots \\ & \ldots + \left[\alpha_k - \alpha_{k-1} \left(\frac{x}{2}\right)^2 + \ldots + (-1)^k \frac{\alpha_0}{(k!)^2} \left(\frac{x}{2}\right)^{2k}\right] \Delta^k q + \ldots. \end{aligned} \tag{8}$$

If into (8) we substitute the condition on the lateral surface of the cylinder

$$x = 1, \quad u = q, \tag{9}$$

we obtain values of $\alpha_k$ from the system of equations

$$\begin{aligned} & \alpha_0 = 1, \\ & \alpha_1 - \alpha_0 \left(\frac{1}{2}\right)^2 = 0, \\ & \alpha_2 - \alpha_1 \left(\frac{1}{2}\right)^2 + \frac{\alpha_0}{(2!)^2} \left(\frac{1}{2}\right)^4 = 0, \\ & \alpha_3 - \alpha_2 \left(\frac{1}{2}\right)^2 + \frac{\alpha_1}{(2!)^2} \left(\frac{1}{2}\right)^4 - \frac{\alpha_0}{(3!)^2} \left(\frac{1}{2}\right)^6 = 0, \\ & \ldots \\ & \alpha_k - \alpha_{k-1} \left(\frac{1}{2}\right)^2 + \ldots + (-1)^k \frac{\alpha_0}{(k!)^2} \left(\frac{1}{2}\right)^{2k} = 0. \\ & \ldots \end{aligned} \tag{10}$$

Hence

$$\alpha_0 = 1, \quad \alpha_1 = \frac{1}{4}, \quad \alpha_2 = \frac{3}{64}, \quad \alpha_3 = \frac{19}{2304} \text{ etc.}$$

The question of the number of terms in the general solution (8) must be decided for each specific problem. It may happen, for example, that to describe the variation of temperature on the lateral surface of a cylinder, a multinomial of seventh degree in z and third degree in $\tau$ will prove sufficient. Then from (3)

$$\Delta^4 q = b^8 \left(\frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial \tau}\right)^4 q = 0 \tag{11}$$

and only the first four terms remain in (8).

It has been shown that series (8) converges even when the number of terms is infinite. Taking some finite sum, we may estimate the error then obtained.

The figure gives curves of variation of the coefficients with $\Delta q$, $\Delta^2 q$ and $\Delta^3 q$ as a function of $x = r/b$.


REFERENCES

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## CHOICE OF NECESSARY THERMAL RESISTANCE OF EXTERIOR WALLS

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In actual buildings, heat transfer between the surrounding medium and the wall structure is unsteady. The method of technical thermal calculation recommended in the code [1] reduces the calculation of unsteady heat transfer to a steady calculation. Such a simplification is a first crude approximation, which may lead to large error under certain conditions.

To calculate unsteady heat transfer through the walls, the code [1] introduces the quantity thermal inertia (characteristic thermal inertia of the wall)

$$D = \sum_{i=1}^{n} R_i s_i = \sum_{i=1}^{n} \frac{\delta_i \omega^{1/2}}{a_i^{1/2}}, \qquad (1)$$

where $s_i$ is the assimilation factor $[s_i = (\lambda_i c_i \gamma_i \omega)^{1/2}]$; $\omega$ is the angular frequency of the temperature oscillation; $R_i = \delta_i/\lambda_i$ is the thermal resistance of the i-th layer.

As has been shown in [2, 3], the quantity D is not a characteristic of the thermal inertia for multilayer and finite thickness structures. Therefore the calculation of an unsteady system and the correction of the calculated outside temperatures according to D in the expression for $R_r$ is unjustified.

According to [1], the insulating value of outside walls must be not less than $R_r$, as determined from the formula

$$R_0^r = \frac{(t_{in} - t_{ex})\, nb}{\alpha_2 \Delta t^d}, \quad \text{or} \quad R_0^r = \frac{(t_{in} - t_{ex})\, nbR_b}{\Delta t^d}, \qquad (2)$$

where $t_{in}$, $t_{ex}$ are the internal and external temperatures, respectively; $\Delta t^d$ is the temperature difference between the calculated temperature of the inside air and the temperature of the inside surface, and n, b are correction coefficients characterizing the location of the structure and the quality of the heat insulation (the advisability of introducing which gives rise to serious objections). Then the R value of the structure is corrected by a whole series of stipulations and recommendations, and it is noted, finally, that R should be increased by a factor of not more than 1.5 in comparison with $R_0^r$ as determined from (2).

Thus, the code procedure for calculating an unsteady system makes use of a whole series of quantities and coefficients, the designation and meaning of which do not make physical sense, namely: D, the thermal inertia, s, Y, thermal assimilation factors, etc.

Solutions in general form were obtained in [4] for multilayer structures with boundary conditions of the first and third kinds and a harmonic temperature variation of the medium. The expression for the temperature on the inside surface has the form

$$t_s = t_{in} + (t_{ex} - t_{in})\frac{1}{\alpha_2 R} + \frac{2^{n-1} K_1 \ldots K_{n-1} \sqrt{2}\, t_m}{(K_1 + 1) \ldots (K_{n-1} + 1)\, \Delta_0^1}, \qquad (3)$$

where $t_{in}$ and $t_{ex}$ are, respectively, the calculated internal and external temperatures; $\alpha_2$ is the heat transfer coefficient on the inside; R is the thermal resistance of the structure; $\frac{2^{n-1} K_1 \ldots K_{n-1} \sqrt{2}}{(K_1 + 1) \ldots (K_{n-1} + 1)\, \Delta_0^1} = k$ is the damping of oscillations in the structure, which, as shown in [2], depends on $Bi_1^*$, $Bi_2^*$, $K_i$, $h_i$, $D_i$ and D; $Bi_1^* = \alpha_1/(\lambda_1 c_1 \gamma_1 \omega)^{1/2}$, $Bi_2^* = \alpha_2/(\lambda_n c_n \gamma_n \omega)^{1/2}$ defines the heat transfer conditions at the boundary and the materials of the outside layers; $K_i = (\lambda_i c_i \gamma_i/\lambda_{i+1} c_{i+1} \gamma_{i+1})^{1/2}$, $h_i = (K_i - 1)/(K_i + 1)$ describes the nonuniformity of the structural material and the order of the layers; $D_i = R_i s_i$ characterizes the ratio of the geometric dimensions of the layers; and $D = \sum_{i=1}^{n} R_i s_i$ is the thermal inertia according to the code [1].

We will use (3) to choose the $R_r$ of wall structures.

The limiting permissible temperature on the inside surface is the dew point temperature $t_\varphi$. Therefore,

$$t_\varphi > t_{in} + (t_{ex} - t_{in})\frac{1}{\alpha_2 R} + kt_m, \qquad (4)$$

in which

$$R > \frac{t_{ex} - t_{in}}{\alpha_2 (t_\varphi - t_{in} + kt_m)}, \qquad (5)$$

or for negative external temperatures

$$R > \frac{t_{in} - t_{ex}}{\alpha_2 (t_{in} - t_\varphi - kt_m)}. \qquad (6)$$

Expression (6), of course, relates the choice of $R_r$ to the calculated external and internal temperatures, the heat transfer conditions, the dew point temperature of the internal air, the amplitude of temperature oscillations of the external air, $t_m$, and the damping of the oscillations, k.

If we calculate the heat flux, using the expression given in [2] for the temperature field, it is divided into two terms, a constant component and a variable component. The ratio of the amount of heat transmitted through the wall due to the variable component (per half period) to the amount transmitted through the wall due to the constant component (per half period) will be termed the nonuniformity of heat loss; as calculations show [3], this quantity varies strongly for different walls.

Therefore, the second requirement to determine the suitability of structures must be the nonuniformity of heat loss (depending on the type and regime of operation of the heaters (or coolers)).

We consider that the choice of $R_r$ should be determined by the following conditions:

a) the R of the structure must not be less than $R_r$, determined from (5) and (6);